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A Comparative Study of Confinement Models

by Shamim A. Sheikh

Various analytical models available in the literature for the confinement of concrete by rectilinear ties are studied. These models are applied to the specimens tested by the author as well as by other investigators to predict the results. Loadings on these specimens include axial and combined axial and bending, monotonic as well as cyclic. In the case of cyclic loading, only the envelope curves are determined using the analytical models. Experimental results are compared with results predicted by various models. It is concluded that in addition to the commonly acknowledged variables such as the amount of lateral reinforcement and steel strength, two other variables play important roles in determining the behavior of the confined concrete. These variables are the distribution of the longitudinal steel around the core perimeter and the resulting tie configuration, and the spacing of ties. Better distribution of steel and closer spacing of ties along the column longitudinal axis (for the same amount of reinforcement) result in higher concrete strength and ductility. Analytical results from the model accounting for these variables show the best agreement with experimental results.

Keywords: axial loads; bending moments; columns (supports); **confined concrete**; ductility; earthquakes; models; reinforced concrete; **stress-strain relationships**; structural analysis; structural design; tied columns; **ties (reinforcement)**.

Circular spirals confine concrete much more effectively than rectilinear ties, and the mechanism of confinement for circular spirals is better understood than for ties. But their relative ease in detailing makes the use of ties more attractive than spirals.

When reinforced concrete sections are subjected to large deformations typical of seismic motions, their ability to carry load depends primarily on the behavior of confined concrete within the core. Numerous studies[1-21] have been reported on the behavior of concrete confined by rectilinear ties. Several analytical models with various degrees of sophistication have been proposed. Some models predict only the ascending part of concrete's stress-strain curve, while others predict the curve up to a certain point on the descending part. A few models predict only concrete strength and corresponding strain.

Several variables have been considered in these models. The amount of lateral reinforcement received most attention. Some of the other variables which appear in these models are strength of plain concrete, steel strength, distribution of longitudinal steel and the resulting tie configuration (steel configuration), tie spacing, and section dimensions.

In this paper the available analytical models[5,8,12,13,15,17,19] are applied to predict the results of tests reported by various investigators.[12,16,17,19,20] The tests include specimens subjected to axial as well as combined axial and bending loads. One set of specimens was tested under constant axial loads and reversal of moments,[20] in which case the envelope moment-curvature curves were calculated and compared with the experimental results. It is generally accepted[21] that the envelope curve under cyclic loading is almost identical to the curve obtained from the monotic loading.

SUMMARY OF MODELS

Almost all the analytical models for confinement are based on experimental results. Most experimental data were obtained from small-scale tests on simple tie configurations. A summary of previous representative tests is given in Table 1. In most of the tests the ratio of the area of the core bounded by the center line of the perimeter tie to the gross area of the specimen was small compared with the values commonly used in practice.

The models proposed by the following researchers are used in this paper: Chan,[5] Kent and Park,[8] Roy and Sozen,[12] Sargin,[13] Sheikh and Uzumeri,[15] Soliman and Yu,[17] and Vallenas, Bertero, and Popov.[19] Variables considered in these models are summarized in Table 2. Direct application of these models to all tests considered in this study was not possible. Therefore, a few assumptions were made to adopt models to tests wherever required. Brief details of these models are given below. The assumptions made for the application of

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these models are also explained. It has been attempted to reduce some notations used by various investigators to the ones used by Sheikh and Uzumeri.[14,15] The increase in concrete strength due to confinement is represented by K_s, the ratio of the strength of confined concrete to the strength of unconfined concrete. The core is defined as the area bounded by the center line of the perimeter tie.

Sheikh and Uzumeri (1980)

In this model[15] the increase in strength of confined concrete was calculated on the basis of effectively confined concrete area, which is less than the core concrete area enclosed by the center line of the perimeter tie. The theoretical-cum-empirical relation used the test results from 24 to 12 in. (305 mm) square, 6 ft 5 in. (1960 mm) high reinforced concrete columns tested under monotonic axial compression. Minimizing the cumulative error for all 24 columns was the only criterion used in determining the empirical constants. No effort was made to minimize the difference between experimental and analytical values for individual columns.

A stress-strain relationship for confined concrete, as shown in Fig. 1(a), was proposed. The first part of the curve, up to a strain of ε_{s1}, is a second degree parabola. Between ε_{s1} and ε_{s2}, the curve has a horizontal straight line portion. Beyond ε_{s2} the descending part of the curve is a straight line which is suggested to continue decreasing to 30 percent of the maximum stress, after which a horizontal line represents the concrete behav-

Table 1 — Summary of available tests

		Details of the specimens				
Reference	Researcher	Number	Size of section, in.	A_{core}/A_{gross}	Longitudinal steel	ϱ_s, percent
9	King (1946)	164	3.5 × 3.5	0.54-0.61	4 corner bars	0.26-7.80
10	King (1946)	18	10 × 10	0.34-0.66	4 corner bars	0-10.56
5	Chan (1955)	9	6 × 6	0.63-0.92	4 corner bars	0.84- 4.13
		7	6 × 3-5/8	0.92-0.96	4 corner bars	0.85- 4.55
		7	6 in. dia.	0.97	4 bars	1.64- 3.28
2	Bresler and	2	8 × 8	0.61	6 bars	0.41- 0.62
	Gilbert (1961)	2	8 × 8	0.61	8 bars	0.43- 0.69
11	Pfister (1964)	4	12 × 12	0.42-0.53	12 bars	0- 0.45
		3	8 × 18	0.36-0.49	12 bars	0- 0.64
		4	10 × 12	0.49	6 bars	0- 0.39
12	Roy and Sozen (1964)	45	5 × 5	0.86-0.9	4 corner bars	2.1 - 2.4
1	Bertero and	2	3 × 3		none	
	Felippa (1964)	5	3 × 3		4 corner bars	0- 2.6
		2	4½ × 4½		none	
		6	4½ × 4½		4 corner bars	
6	Hudson (1966)	32	4 × 4	0.46-0.47	8 bars	0- 0.32
		28	6 × 6	0.63-0.66	8 bars	0- 0.69
17	Soliman and Yu	3	6 × 4	0.92-1.00	2 bars	0- 0.40
	(1967)	11	6 × 4	0.44-0.92	4 corner bars	0.6 - 3.43
		1	6 × 3	0.91	4 corner bars	1.46
		1	6 × 5	0.93	4 corner bars	1.05
14	Shah and Rangan (1970)	11	2 × 2	0.83	none	0- 1.01
18	Somes (1970)	42	4 × 4	0.88-0.92	none	0.67- 9.04
13	Sargin (1971)	41	5 × 5	0.65-0.96	none	0.59- 5.32
4	Burdette and	16	5 × 5	0.72-1.00	none	0- 3.70
	Hilsdorf (1971)	4	5 in. dia.	1.00	none	1.85- 6.68
3	Bunni (1975)	4	5 × 5	0.88-0.90	none	0
		50	5 × 5	0.88-0.95	4 corner bars	0- 6.55
7	Kaar et al.	11	10 × 16	0.68-0.72	4 corner bars	0.96- 3.86
	(1977)	6	5 × 8	0.70	4 corner bars	0 or 1.72
19	Vallenas,	3	10 × 10	0.78	8 bars	
	Bertero and	3	9 × 9	0.96	8 bars	1.40
	Popov (1977)	3	10 × 10	0.78	none	
		3	9 × 9	0.96	none	
16	Sheikh and	9	12 × 12	0.78	8 bars	0.80- 2.32
	Uzumeri (1980)	6	12 × 12	0.78	12 bars	1.60- 2.40
		9	12 × 12	0.78	16 bars	0.76- 2.30
21	Scott et al.	8	17.7 × 17.7	0.79-0.80	8 bars	1.34- 2.93
	(1982)	12	17.7 × 17.7	0.79-0.80	12 bars	1.40- 3.09

1 in. = 25.4 mm.

Table 2 — List of variables for different models

Model	Variables
Sheikh and Uzumeri[15]	Volumetric ratio of lateral steel to concrete core, distribution of longitudinal steel around the core perimeter and the resulting tie configuration, tie spacing, characteristics of lateral steel, and strength of plain concrete.
Chan[5]	Volumetric ratio of lateral steel to concrete core.
Roy and Sozen[12]	Volumetric ratio of lateral steel to concrete core and ratio of section dimension to tie spacing.
Soliman and Yu[17]	Area of tie steel bar, tie spacing, and section geometry.
Sargin[13]	Volumetric ratio of lateral steel to concrete core, ratio of width of concrete core to tie spacing, steel strength, and strength of plain concrete.
Kent and Park[18]	Volumetric ratio of lateral steel to concrete core, ratio of width of concrete core to tie spacing and strength of plain concrete.
Vallenas, Bertero, and Popov[19]	Volumetric ratio of lateral steel to concrete core, ratio of area of longitudinal steel to the area of cross section, sizes of tie bar and longitudinal bar, ratio of core dimension to tie spacing, steel strength, and strength of plain concrete.

ior. Four equations were given to define the curve completely. The four values are K_s, ratio of the strength of confined concrete to the strength of unconfined concrete; ε_{s1} and ε_{s2}, the minimum and maximum strains corresponding to the maximum stress in concrete; and ε_{s85}, the strain corresponding to 85 percent of the maximum stress on the descending part of the curve.

The following variables were considered in the development of this model: volumetric ratio of lateral reinforcement to concrete core; distribution of longitudinal steel around the core perimeter and the resulting tie configuration; tie spacing; characteristics of lateral steel; and strength of plain concrete. The amount of longitudinal steel was recognized as having no significant effect on the behavior of confined concrete.

Chan (1955)

Chan's[5] equations to predict the strength of confined concrete and the corresponding strain were based on tests in which load was applied with a small eccentricity on 6 × 6 × 11½ in. (152 × 152 × 292 mm) and 6 × 3⅝ × 52 in. (152 × 92 × 1321 mm) specimens. Two equations, one for K_u/K_o, equal to K_s, and the other for ε_u, the ultimate strain in the column when concrete carries the maximum load, were suggested as functions of the volumetric ratio of tie steel to concrete core. No other variables were believed to affect the strength of confined concrete and the corresponding strain value.

The suggested trilinear curve for unconfined and confined concrete is shown in Fig. 1(b). OAB approximates the curve for unconfined concrete. In plain concrete, slope of BC is negative. In confined concrete, with suitable lateral binding, λ_2 can be positive, with ε_u attaining values much greater than for unconfined concrete.

Roy and Sozen (1964)

Tests[12] on 5 × 5 × 25 in. (127 × 127 × 635 mm) prisms led Roy and Sozen to conclude that the confinement provided by rectilinear ties does not enhance the strength of the confined concrete. They proposed a stress-strain relationship of concrete, shown in Fig. 1(c), in which the coordinates of the peak point are f_p, .002, where f_p is the strength of concrete in a plain specimen. This means K_s = 1.0. An equation for the strain value corresponding to 50 percent of the maximum stress was suggested to define the descending part of the bilinear curve. The only variables considered to affect the ductility of the confined concrete are the volumetric ratio of tie steel to concrete core and the ratio of the shorter side dimension of the compressed concrete section to the tie spacing.

Soliman and Yu (1967)

A stress-strain relation for confined concrete was proposed[17] in which four points were defined [Fig. 1(d)]. Four equations were empirically developed using the data obtained from eccentric compression tests on 3 × 6 in. (76 × 152 mm), 4 × 6 in. (102 × 152 mm) and 5 × 6 in. (127 × 152) specimens. The initial part of the proposed relation consists of a parabolic curve with the peak value of f_{cc}, ε_{ce}. The second part of the relation is a horizontal straight line up to a strain value of ε_{cs}. The final part is a straight line with a negative slope representing the descending branch. This was defined by a strain value ε_{cf} corresponding to 80 percent of the maximum stress.

All test specimens used simple tie arrangements with only one tie at one level. The variables considered were area of tie steel bar, tie spacing, and section geometry. For the purpose of applying the proposed equations to the tests with complex tie configurations, the lateral steel is assumed to be distributed around the core perimeter. This assumption is necessary to calculate a reasonable value of area of tie bar to be used in the proposed equations.

Sargin (1971)

Three equations were proposed to predict the ultimate strength of confined concrete and one to predict the corresponding strain values ε_{oc}.[13] The empirical equations are based on regression analysis of results of tests conducted on 5 × 5 × 25 in. (127 × 127 × 635 mm) specimens under concentric and eccentric compression loads. As observed previously by Hognestad et al.,[22] Sargin also found no significant difference in strength between concentrically and eccentrically loaded specimens. The following variables are recognized in Sargin's equations: volumetric ratio of lateral reinforcement to concrete core, ratio of tie spacing to the width of concrete core, yield strength of steel, and concrete cylinder strength. The strain value ε_{oc} at peak stress is also assumed to depend on the strain gradient at the section in addition to the above variables. A general equation was proposed to give a continuous stress-strain curve of confined concrete as shown in Fig. 1(e).

Kent and Park (1971)

On the basis of existing experimental evidence, Kent and Park proposed the stress-strain curve shown in Fig. 1(e) for concrete confined by rectilinear ties. The suggested curve combined many features of previously proposed curves.

The ascending part of the proposed curve is unaffected by confinement. Peak stress and strain values are given as f_c' and .002. The falling branch of the curve is suggested to be a straight line whose slope is a function of concrete cylinder strength, ratio of width of confined concrete to spacing of ties, and ratio of volume of tie steel to volume of concrete core. The descending part of the curve extends to 0.2 f_c', beyond which a horizontal line represents the concrete behavior. The data reported by Roy and Sozen,[12] Bertero and Felipa,[1] and Soliman and Yu[17] were used in developing the proposed curve.

This model suggests that confinement due to rectilinear ties does not enhance concrete strength, which means $K_s = 1.0$. The effect of confinement on ductility is recognized.

Vallenas, Bertero, Popov (1977)

The stress-strain curve proposed by Vallenas et al.[19] [Fig. 1(g)] is similar in form to Kent and Park's model,[8] shown in Fig. 1(f). The main difference between the two is the inclusion of concrete strength enhancement due to confinement in the model proposed by Vallenas et al. The ascending part of the curve is represented by a second degree parabola. The descending branch of the curve consists of a straight line with a given slope extending to 30 percent of the maximum stress, beyond which the curve continues in the form of a horizontal straight line. The following variables are included in the model: volumetric ratio of lateral steel to concrete core, ratio of area of longitudinal steel to the area of cross section, sizes of tie bar and longitudinal bar, ratio of tie spacing to core dimension, concrete strength, and strength of tie steel. A major difference between this model and the one proposed by Sheikh and Uzumeri[15] is that in the Vallenas model strength enhancement of confined concrete is considered proportional to the volumetric ratio of longitudinal steel to concrete, whereas in the Sheikh and Uzumeri model strength enhancement depends upon distribution of the longitudinal steel.

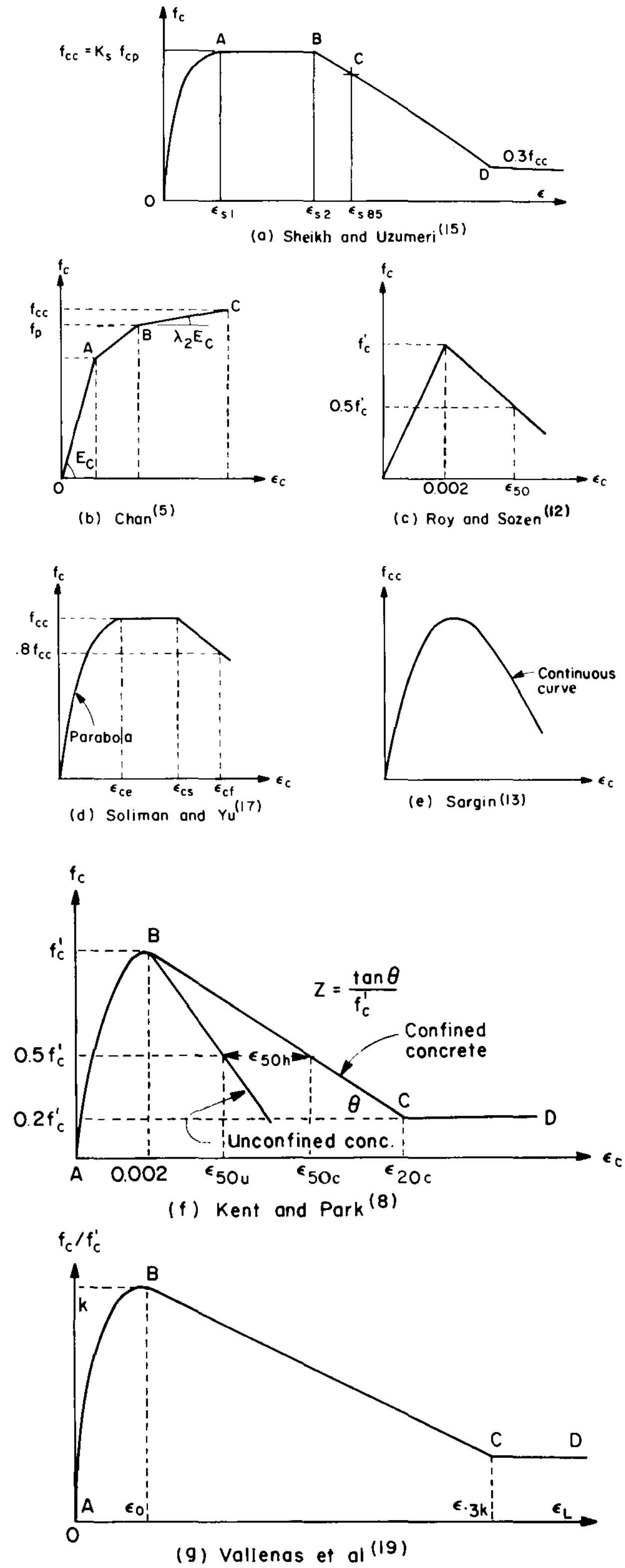

Fig. 1 — Some proposed stress-strain curves for concrete confined by rectilinear ties

APPLICATION OF MODELS

The seven confinement models discussed above were used to predict the results of tests reported by various

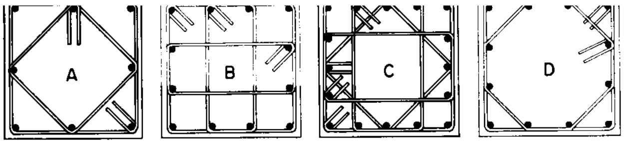

Fig. 2 — Various steel configurations

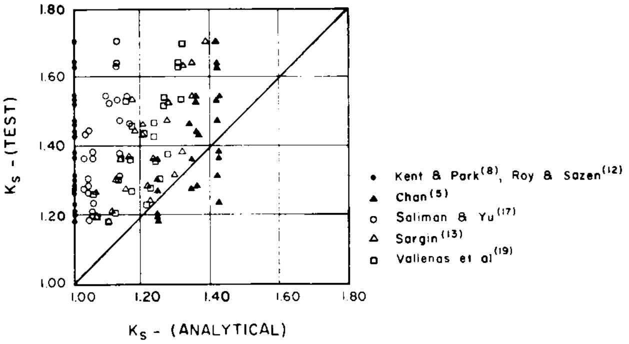

Fig. 3 — Comparison of experimental K_s *values with the* K_s *values calculated from various models for 24 columns*

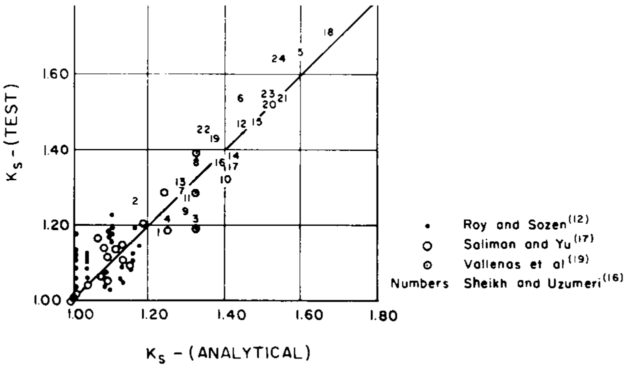

Fig. 4 — Comparison between experimental K_s *values for various tests and* K_s *values calculated from the model proposed by Sheikh and Uzumeri*

researchers. These tests included specimens under monotonic axial compression and eccentric compression. One set of specimens was tested under axial load and reversed cyclic bending, in which case it was attempted to predict the moment-curvature envelope curves from the analytical models.

Monotonic Compression

The results of tests reported by Roy and Sozen,[12] Soliman and Yu,[17] Vallenas et al.,[19] and Scott et al.[21] were predicted using the Sheikh and Uzumeri model. All seven models were applied to predict the results of tests reported by Sheikh and Uzumeri[16] in which a large variety of test variables were studied. These variables could be isolated by dividing the 24 specimens into subsets where only one factor varyed between different specimens. Various arrangements of longitudinal and lateral reinforcements, tie spacing, amount of longitudinal and lateral steels, and characteristics of steel were studied for their effects on the behavior of confined concrete. The amount of longitudinal steel was found to have no significant effect on properties of confined concrete. The four steel configurations tested are shown in Fig. 2. The large size of these columns, appropriate ratios of the area of core to gross area of the specimens, and complex steel arrangements resulted in large increases in concrete strength (up to 70 percent) and ductility due to confinement. In most other tests reported in the literature, small increase in concrete strength was observed due to rectilinear confinement. It has been demonstrated[15,16] that in specimens with only four corner bars, the effectively confined concrete area at the critical section between the ties would be very small compared with the core area bounded by the center line of the tie. This will result in poor confinement of the core concrete. Small specimen sizes, simple steel arrangements, low volumetric ratios of lateral steel to concrete core, and low ratios of core area to gross area of columns would result in only small increases in confined concrete strength and ductility. Checking the models against these test results will not distinguish between the accuracies of various models. A comprehensive study of the models is, therefore, based on test results reported by Sheikh and Uzumeri.[16]

The significant values related to concrete behavior were determined for all 24 columns, using the seven models. The experimental and analytical strength values, represented by K_s, are compared in Fig. 3 for six models. The analytical K_s values calculated using the model proposed by Sheikh and Uzumeri[15] are compared with the experimental K_s values in Fig. 4. The strength of unconfined concrete in the columns was observed to be equal to 0.85 f_c'.

The analytical curves from some of the models are compared with the experimental curves in Fig. 5 for nine representative columns, which include all four steel configurations. Size of the tie bar, spacing of ties, volumetric ratio of tie steel to concrete core ϱ_s, ratio of longitudinal steel area to gross cross-section area ϱ, and arrangement of longitudinal and lateral steel are also given in Fig. 5 along with the curves for each column. The models proposed by Kent and Park,[8] Roy and Sozen,[12] Sheikh and Uzumeri,[15] Soliman and Yu,[17] and Vallenas et al.[19] were used for this comparison. Comparisons between experimental and analytical curves are presented for only nine columns. A very similar trend is observed in the other 15 columns, as can be seen in Fig. 6 where areas under experimental curves are compared with areas under analytical curves. For all the columns, these areas are calculated up to the strain values at which the experimental curves terminate. In the case of a few columns, tests were terminated somewhat prematurely, for fear of damaging the equipment. For other columns, tests were terminated when either the load suddenly dropped to a low fraction of the maximum, resulting in a complete destruction of the specimen, or no instrument was left in

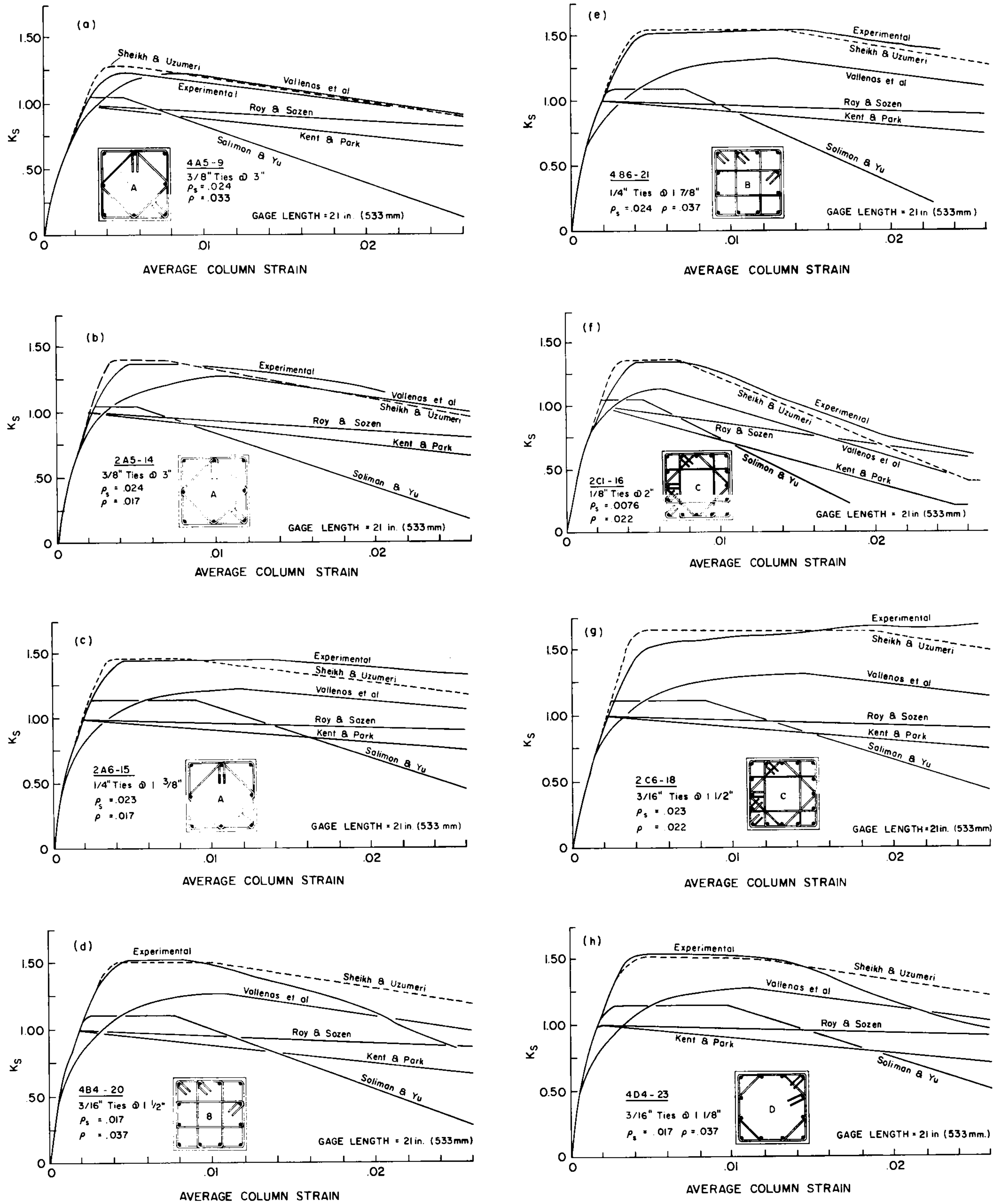

Fig. 5 — Comparison of experimental and analytical stress-strain curves

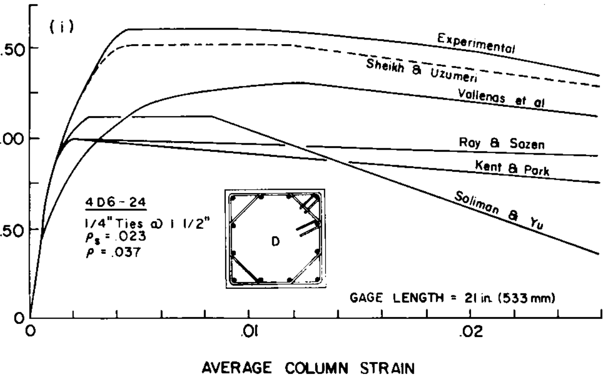

Fig. 5 (cont.) — Comparison of experimental and analytical stress-strain curves

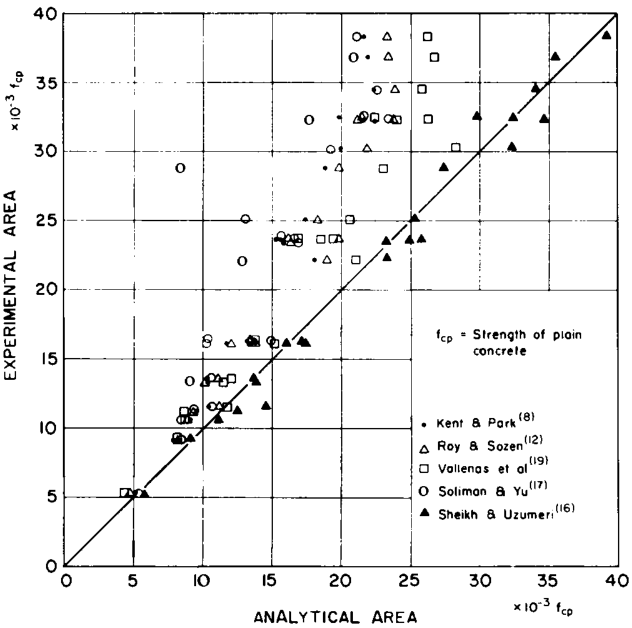

Fig. 6 — Comparison between areas under the experimental stress-strain curves for 24 columns and the corresponding analytical areas calculated using various models

position to give further readings. Therefore, the areas under the curves plotted in Fig. 6 do not necessarily indicate the relative toughness of the columns.

Comparing experimental and analytical results shown in Fig. 3, 4, 5, and 6 with the nature of the equations, the following points can be made about various models.

Chan's equation[5] overestimates the strength increase of confined concrete in several cases, particularly for the specimens of Configuration A and those with large tie spacings. As no consideration is given to tie spacing and steel configuration, the equation underestimates concrete strength for columns with tightly knit cages. Similar comments can be made about the equation predicting the ultimate strain and hence ductility of confined concrete.

The models proposed by Roy and Sozen[12] and Kent and Park[8] do not recognize any strength increase in confined concrete. Therefore, a complete lack of agreement is found in Fig. 3 between experimental and analytical K_s values. A strain of .002 at maximum stress in concrete, as suggested by the two models, underestimates the corresponding experimental strain values in all the columns. The slope of the descending part of the curve is reasonably estimated by Kent and Park's model in most columns, but the descending parts of Roy and Sozen's curves show a slower rate of drop in stress than shown by experimental curves in many cases, particularly in columns with small tie spacings. The energy absorbed by the columns up to certain strain values, as represented by the area under the curves, is underestimated by both models, as shown in Fig. 6, mainly because of lower predicted concrete strength.

The increased concrete strength due to confinement is underestimated for all 24 columns by the model proposed by Soliman and Yu.[17] The difference between the experimental and analytical K_s values increases with the increase in the number of laterally supported longitudinal bars. This is because no consideration is given to steel configuration in the model. Except for columns with C configurations and high volumetric ratio of lateral steel to concrete core, the model predicts the maximum strain corresponding to the maximum stress reasonably well. The descending parts of the curves are too steep compared with the experimental curves for almost all the columns. This results in the lowest predicted values of energy absorbed by the columns, as shown in Fig. 6.

Close agreement is found between K_s values given by Sargin's three proposed equations for most of the columns. Compared to the experimental values, these equations underestimate the strength of confined concrete for most of the columns. The difference between experimental and calculated values is higher for the columns of Configurations B, C, and D than for Type A columns. The effect of tie spacing, it seems, is not appropriately accounted for in Sargin's equations. Increase in concrete strength is considered to be directly proportional to the stress in tie steel. The experimental data does not support this assumption.

The equation proposed by Vallenas et al.[19] underestimates the strength of confined concrete in all columns. The difference between experimental and analytical strength factors K_s increases with increase in the number of laterally supported longitudinal bars. Experimental data does not support the assumption that strength of confined concrete is dependent upon longitudinal steel content. This is clearly demonstrated from comparison of experimental and analytical curves in Fig. 5(a) and (b). The model predicts the results for Column 4A5-9 (ϱ = .033) quite well, but the prediction does not remain as accurate for Column 2A5-14, in which the longitudinal steel content (ϱ) is reduced to .017. An examination of the experimental and the analytical curves for Columns 2A5-14 and 2A6-15 shows that the effect of tie spacing is more pronounced than recognized by this model. This observation was also

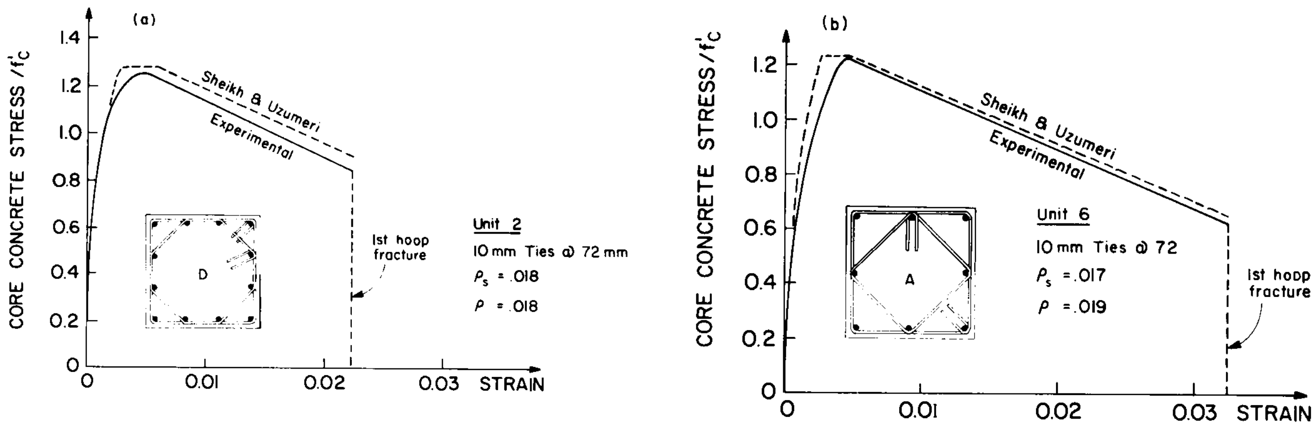

Fig. 7 — Experimental and analytical core concrete stress-strain curves for specimens tested by Scott et al.[21]

made in several other pairs of columns. Experimental evidence shows that increase in concrete strength is not directly proportional to the stress in lateral steel, as given by the equation. The slope of the descending part of the curve is the same as given by Kent and Park,[8] but due to higher calculated strain values corresponding to the maximum stress, the strain characteristics of most columns are better predicted by this model. With the exception of the model proposed by Sheikh and Uzumeri,[15] this model predicted the results of the tests better than other models studied here (Fig. 5 and 6).

The analytical behaviors of confined concrete as calculated using the model proposed by Sheikh and Uzumeri[15] show a close agreement with the experimental results for all 24 columns. In addition to the 24 tests, the proposed model was used to predict confined concrete strength for tests reported by various other researchers.[12,17,19] Experimental and analytical K_s values for these tests are also compared in Fig. 4. It is apparent that most of the previous tests were done in low ranges of K_s values. Simple tie arrangements with only four corner bars, low ratios of core area to gross area of the specimens, and small-scale specimens result in small increase in concrete strength. Therefore, the total capacity of the specimen, after the cover had spalled off, would not exceed the unconfined specimen's capacity. This seems to be the main reason for the disagreement among researchers about the increase in strength of concrete confined by rectilinear ties. The proposed model was also applied to predict complete behavior of two specimens recently reported by Scott et al.[21] The 450 mm square columns were tested under concentric loading at a strain rate of 3.3×10^{-6} per sec. Fig. 7 shows the comparison of the experimental and predicted curves. Considering the scatter in the experimental results, the performance of the model is quite satisfactory.

Axial load and cyclic flexure

The four models, Kent and Park,[8] Roy and Sozen[12] Sheikh and Uzumeri,[15] and Vallenas et al.[19] were used to predict the results of tests conducted at the University of Canterbury, New Zealand.[20] Four specimens 550 mm (21.6 in.) square and 3300 mm (10 ft 10 in.) long were tested under combined axial and bending loads. Each specimen was subjected to a cyclic lateral load sequence, while the axial load remained constant throughout the test. Ratios $P_e/A_g f_c'$ for four specimens were 0.21, 0.26, 0.42, and 0.60, where P_e is the axial load and A_g is the gross area of cross section of column.

Since confinement comes into effect only at large strains, moment capacities of the sections were calculated mostly at relatively high curvature values. Extreme fiber compressive strains of .004 or larger were used for the concrete core. Iterative procedure was used to find the depth of the neutral axis required for the internal forces to balance the external applied load. Cover concrete was assumed to be effective up to a strain of ε_{50u} (the strain corresponding to 50 percent of the maximum stress in cylinder on the descending part of the curve) and to be lost at higher strains. The stress-strain curve proposed by Kent and Park[8] for unconfined concrete was used along with the other models to calculate cover concrete's contribution. The moments were calculated about the plastic centroid of the original section.

Fig. 8 shows a comparison of experimental and analytical moment values when the extreme fiber compressive strain for the concrete core was .005. The predictions from the model by Sheikh and Uzumeri[15] remain within 4 percent for all four specimens. At lower levels of axial load, all four models give fairly accurate results. At higher levels of axial load, only the model by Sheikh and Uzumeri remains accurate.

Fig. 9 (a) to (d) shows the experimental moment-curvature behaviors of the four specimens[20] under cyclic loading. The envelope moment-curvature curves as determined from four analytical models are also shown for each specimen. The section capacity calculated according to the ACI Code[2] is also given. In all four specimens, the model by Sheikh and Uzumeri gives results consistently better than other available methods. However, in all cases predictions are conservative. It appears that the strain gradient in the column section results in additional confinement of concrete which seems to be responsible for slightly increased moment capacity of the section. The difference between the pre-

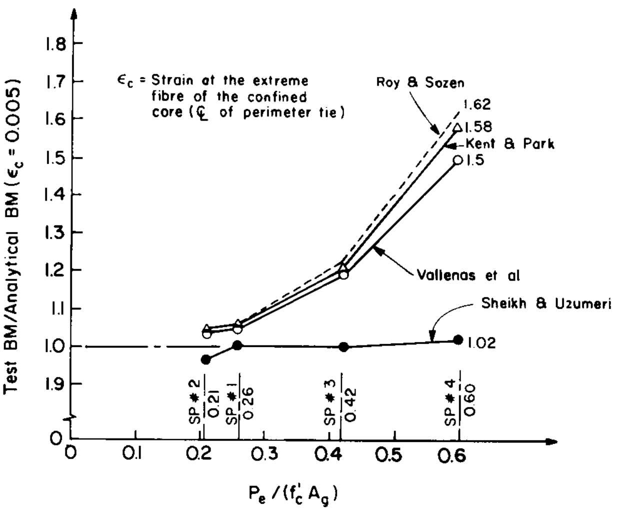

Fig. 8 — Comparison between experimental moment values and moment values calculated from various models for four specimens under different axial loads

dicted and experimental moment capacities, however, is quite small. Studies of the effects of strain gradient on concrete behavior have shown varied results.[8,13,23,25,26] Sturman et al.[26] found in their tests that the peak of the flexural stress-strain curve was located at a strain about 50 percent higher and at a stress about 20 percent larger than the peak of the curve for concentric compression. Ford et al.[25] and Kent and Park,[8] however, indicated that the presence of a transverse strain gradient is not a significant parameter. It is evident that more experimental data is required to investigate the effects of strain gradient on the behavior of confined concrete. This should include specimens with various arrangements of longitudinal and lateral steel under various levels of axial load. In the meantime, the monotonic stress-strain relationship for concrete confined by rectilinear ties, proposed by Sheikh and Uzumeri,[15] can be used to conservatively determine the envelope moment-curvature curve for members under cyclic bending with reasonable accuracy.

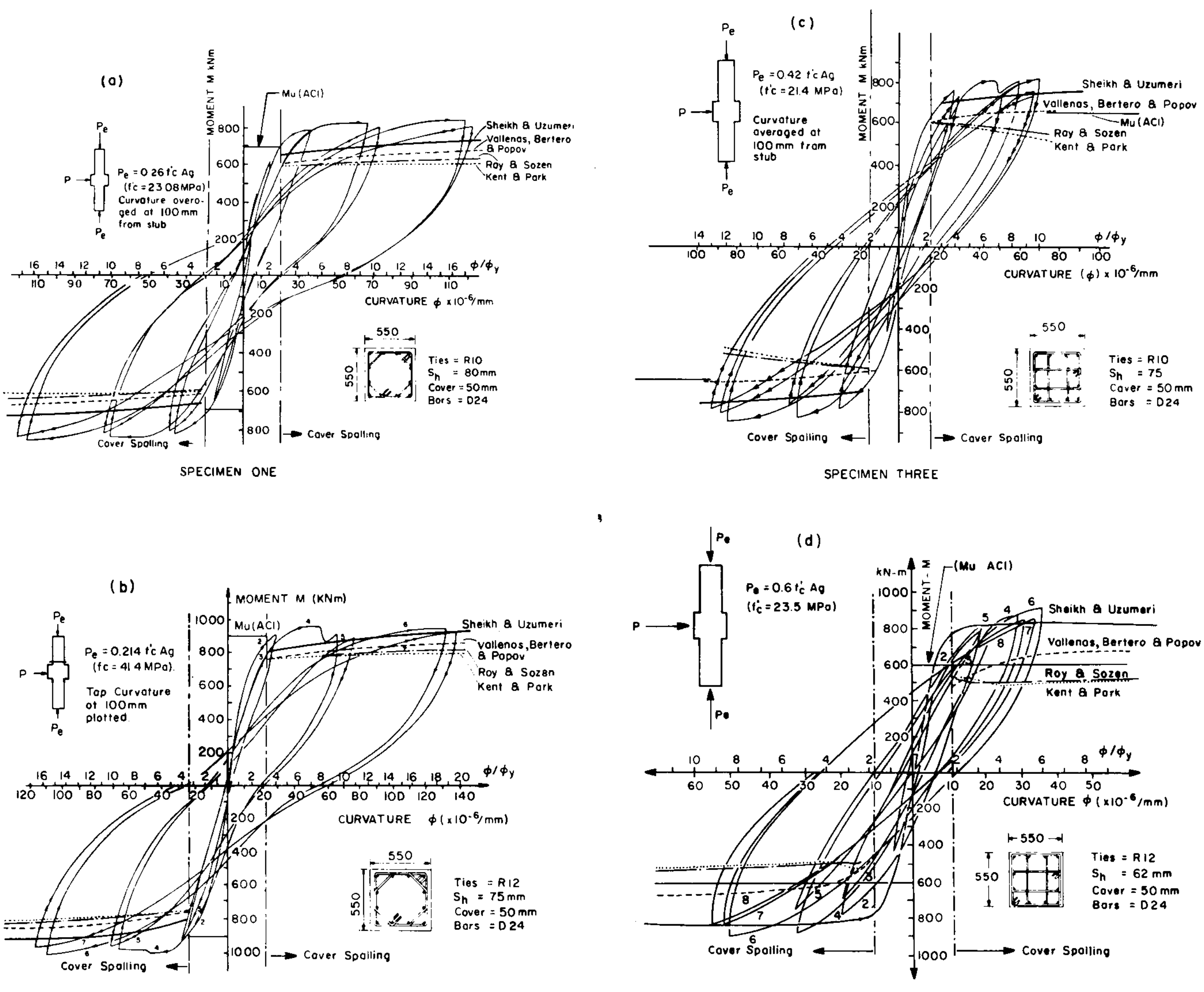

Fig. 9 — Prediction of envelope moment-curvature relationships for specimens under cyclic bending

SUMMARY AND CONCLUSIONS

Analytical models for concrete confinement by rectilinear reinforcement proposed by various researchers were studied. Seven models were applied to specimens tested by the author and by other investigators. The specimens were tested under axial loads only, as well as under the combined effects of axial and flexural loads. Analytical evaluation of the envelope curves for a set of specimens tested under fixed axial loads and cyclic bending was also included in this study. The following conclusions can be drawn from this work.

The model proposed by Sheikh and Uzumeri[15] predicts test results better than the other models studied, both for axial load only and for combined axial and flexural loads. This is the only model that considers the distribution of longitudinal steel and the resulting tie configurations as a variable affecting the mechanism of confinement. Experimental results confirm the beneficial effects of better distribution of longitudinal and lateral steel. Volumetric ratio of lateral steel to concrete core is considered in all seven models as a significant variable affecting concrete behavior. Other variables affecting concrete behavior are tie spacing, section dimensions, cylinder strength, and steel strength. From the limited work reported here, it can also be concluded that the envelope moment-curvature curve for reinforced concrete section under cyclic bending can be determined with reasonable accuracy by using Sheikh and Uzumeri's stress-strain relationship for confined concrete.

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NOTATION

A_g = gross area of cross section of column

f_c' = strength of plain concrete as measured from a standard cylinder (6 × 12 in.) test

f_{cc} = strength of confined concrete

f_{cp}, f_p = strength of concrete in plain specimen

K_u = ratio of confined concrete strength in a column to strength of plain concrete in a standard specimen

K_O = ratio of plain concrete strength in a column to plain concrete strength in a standard specimen

K_s = ratio of confined concrete strength to plain concrete strength in the specimen of similar size and shape = f_{cc}/f_{cp}

P_e = axial load on specimen

ε_{ce}, ε_{cf}, ε_{cs} = concrete strain values[17] [Fig. 1(d)]

ε_{oc} = average longitudinal strain corresponding to the maximum stress in concrete

ε_{s1}, ε_{s2} = minimum and maximum average longitudinal strain corresponding to the maximum stress in concrete [Fig. 1(a)]

ε_{s85} = average longitudinal strain corresponding to 85 percent of the maximum stress in concrete on the unloading part of the curve [Fig. 1(a)]

ε_u = ultimate strain in the column when concrete carries the maximum load[5]

ϱ = ratio of the area of longitudinal steel to the cross-sectional area of the column

ϱ_s = ratio of the volume of lateral steel to the volume of concrete core

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